Stratified flows with variable density: mathematical modelling and numerical challenges.

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Abstract

Stratified flows appear in a wide variety of fundamental problems in hydrological and geophysical sciences. They may involve from hyperconcentrated floods carrying sediment causing collapse, landslides and debris flows, to suspended material in turbidity currents where turbulence is a key process. Also, in stratified flows variable horizontal density is present. Depending on the case, density varies according to the volumetric concentration of different components or species that can represent transported or suspended materials or soluble substances. Multilayer approaches based on the shallow water equations provide suitable models but are not free from difficulties when moving to the numerical resolution of the governing equations. Considering the variety of temporal and spatial scales, transfer of mass and energy among layers may strongly differ from one case to another. As a consequence, in order to provide accurate solutions, very high order methods of proved quality are demanded. Under these complex scenarios it is necessary to observe that the numerical solution provides the expected order of accuracy but also converges to the physically based solution, which is not an easy task. A 2D case that includes interaction with obstacles illustrates the stability and robustness of the numerical scheme in presence of non-uniform density and wetting/drying fronts.

1. Introduction

When we solve the two-layer system, several difficulties arise. First, the system is conditionally hyperbolic. If the difference of the velocities of two layers becomes large enough, then the system loses hyperbolicity, and we expect Kelvin-Helmholtz instability. Secondly, we cannot find the explicit expression for the eigenvalues of the two layer system. Also we need a numerical scheme which is well-balancing with source term since the system is non-conserve. The two-layer shallow water system is accepted as numerical model not only for the flows with different densities but also for the tsunamis generated by underwater landslides.

2. Mathematical model

The model involves the following assumptions: (i) shallow-water approach: the flow is oriented in a predominantly horizontal direction and is confined to a layer which is thin compared to the horizontal scale of interest; (ii) the mixture of water and sediments is described by using the continuum approach and assuming the same velocity for the liquid and for the solid/dissolved phases; (iii) morphodynamic interface: the bed boundary Γ_b is fixed in time and (iv) variable density in each layer.

Accordingly, ϕ_p represents the scalar depth-averaged volumetric concentration of component p, with $p = 1, ..., N_p$ and N_p the number of different components transported. The mixture density is given by $\rho_w r$ where ρ_w is the density of the water and r represents the relative density of the bulk mixture to that of the clean water

$$r = 1 + \sum_{p=1}^{N_p} \Delta_p \phi_p \tag{1}$$

where $\Delta_p = (\rho_p - \rho_w)/\rho_w$ is the relative buoyant density of the solid phase p. We assume that dissolved species with low concentration do not change bulk density $\Delta_p = 0$.

The relevant formulation of the model derives, respectively, from the depth-averaged equation of bulk mass conservation, mixture momentum conservation and conservation of the mass of the different constituents. That system of partial differential equations is used here in coupled form.

3. Applications

The performance of the Reduced Godunov scheme for multicomponent flow is analyzed by means of two dam break numerical experiments in a rectangular tank, 7 m long and 4m wide. The tank bed include two obstacles. Both dam break numerical test share the same initial apparent density in both layers are presented here, different among layers, due to the difference in the concentration of solid/dissolved phases, always much larger in the lower layer. Initial conditions also involve variations in the density of each layer. In test 1 Δ_p is variable for three different components, with Δ_1=1.30, Δ_1=1.40, Δ_1=1.60 while volumetric concentration is set constant in each region defining the dam-break problem. In test 2 Δ_1=Δ_1=Δ_1=1.65 is constant for the three different components but concentration varies in each transported material following a sinusoidal pattern.

In order to provide a more realistic mathematical formulation of shallow flows in environmental analysis, a conservative and robust numerical scheme able to handle with complex situations is used. The system of equations is formed by the 2D shallow water equations, retaining horizontal density variation for mass and momentum of the mixture, supplemented by supplemented by equations for the mass or volume fraction of the mixture constituents. The original Roe's average values for clean water have been extended to include the variation of density, written in terms of volumetric concentration of the different constituents. The resulting approximate Jacobian matrix satisfies the consistency condition, that is a prior requirement when defining approximate solvers.

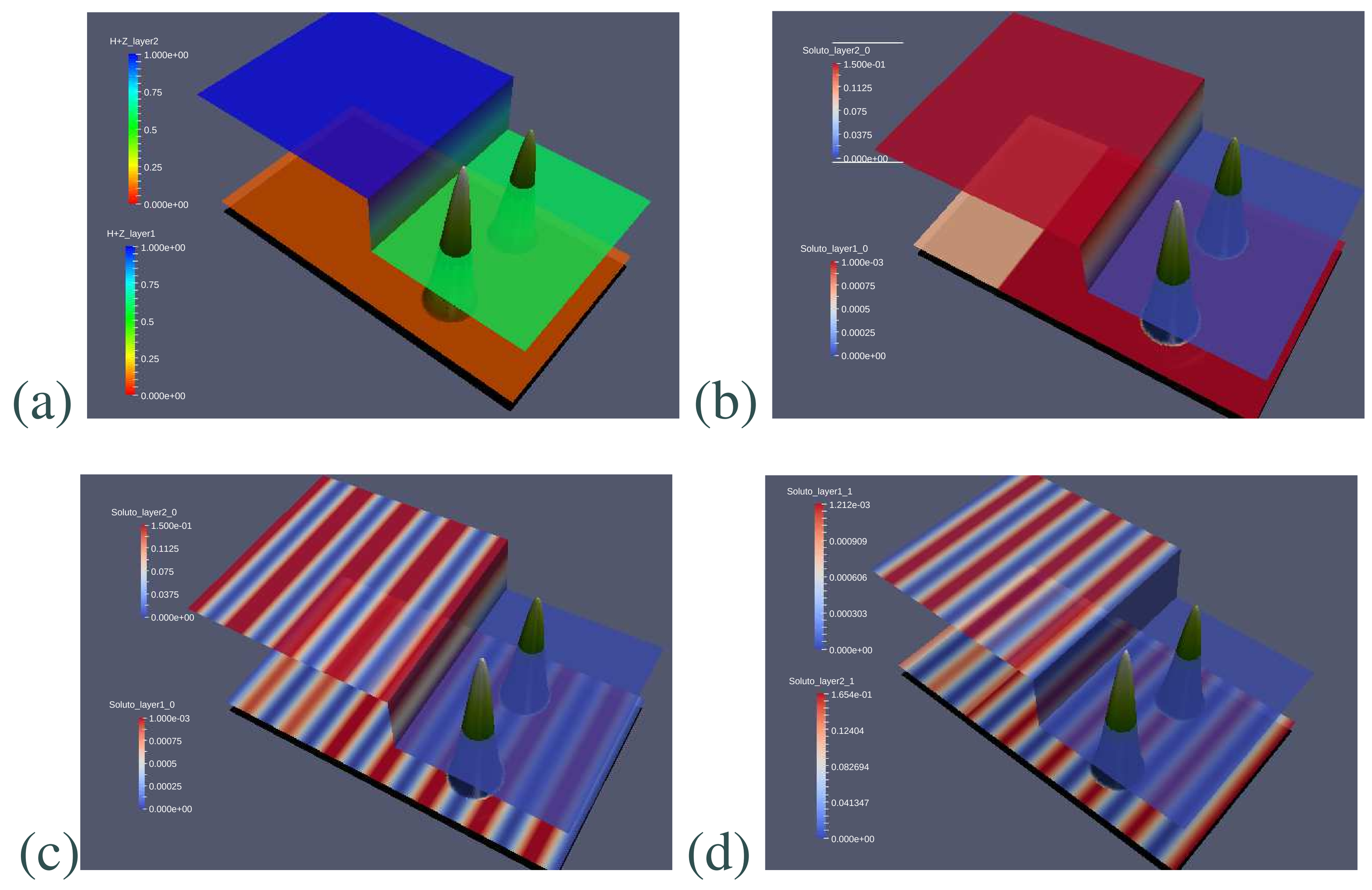

Figure 1: Initial condition for at $t = 0$ s (a) for water level surface, (b) volumetric concentration $\phi_{1,1}$ and $\phi_{1,2}$ for layers 1 and 2 in test 1, (c) volumetric concentration $\phi_{1,1}$ and $\phi_{1,2}$ for layers 1 and 2 in test 2 and (d) volumetric concentration $\phi_{2,1}$ and $\phi_{2,2}$ for layers 1 and 2 in test 2.

The results show that the initial discontinuities produce a complex pattern of shocks and rarefactions that interact with the bed irregularities following the same tendency on both grids. The waves travel at the same speed and the areas covered or uncovered by the wetting/drying fronts are almost identical in both test cases. The observed surface waves are similar in both cases. Initial conditions in the concentration have not a great influence on the velocity fields obtained, that are very similar. Numerical diffusion linked to transport is the responsible for this difference, and may be reduced by using high order. The numerical results show, how, even using different but equivalent initial concentrations, the numerical scheme is able to reproduce the same results, involving wet/dry fronts for both coupled layers.

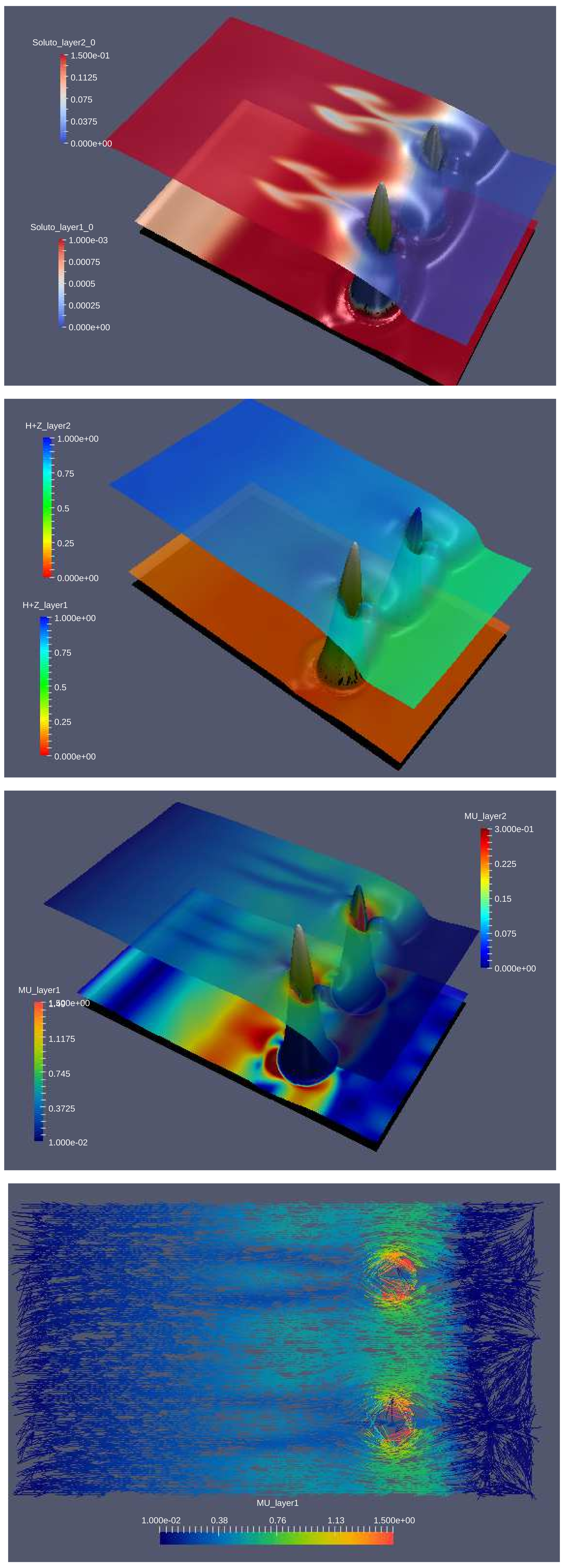

Figure 1: Test case 1. From upper to lower: numerical solution at $t = 6$ s for volumetric concentration $\phi_{1,1}$ and $\phi_{1,2}$ for layers 1 and 2, water surface elevation for layers 1 and 2, module of the velocity for layers 1 and 2 and vector velocity map for layer 1.

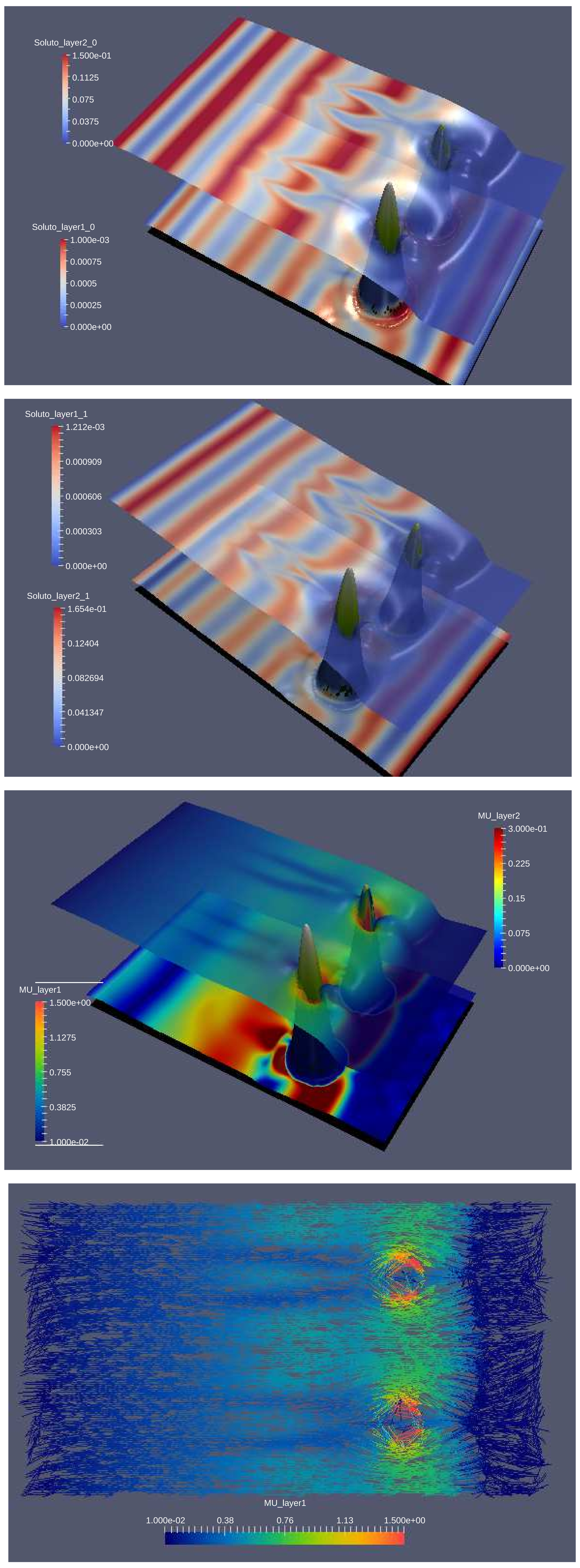

Figure 2: Test case 1. From upper to lower: numerical solution at $t = 6$ s for volumetric concentration $\phi_{1,1}$ in layer 1 and $\phi_{1,2}$ for layer 2, volumetric concentration $\phi_{2,1}$ in layer 1 and $\phi_{2,2}$ for layer 2, module of the velocity for layers 1 and 2 and vector velocity map for layer 1.

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